THE RADIATION OF A HEAT-CONDUCTING ROD IN A VACUUM

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We consider the solution of the problem dealing with the radiation of a thin long rod in a vacuum. It is demonstrated that under the conditions of a quasi-steady regime (given boundary conditions of the first to third kind) it is possible to avoid the operation of an exact solution for the differential equation.

Let us consider a system consisting of a thin long rod which is fastened to a solid body at one end and placed inside a chamber from which the air has been evacuated. The vacuum is such that there is no convective heat transfer between the surface of the rod and the medium. A certain quantity of heat Q is transferred to the rod from the solid body and this heat is transmitted along the rod by means of conduction and removed from the surface through radiation.

We will assume that the temperature fields over the cross section of the rod are constant and that the temperature distribution along the axis of the rod is independent of time, i.e., it is treated as a steady regime.

Let T_0 denote the temperature at the initial cross section of the rod ($x = 0$), and let T_l denote the temperature at the final cross section ($x = l$), and let T_2 denote the temperature of the chamber walls. Moreover, in addition, c_1 will denote the radiation from the lateral surface of the rod, while the reference coefficient for the rod-chamber system is denoted c_{ref} [1–3]

$$c_{ref} = 1 \Big/ \left[\frac{1}{c_1} + \frac{F_1}{F_2}\left(\frac{1}{c_2} - \frac{1}{c_0}\right)\right].$$

When $F_2 \gg F_1$, we will have $c_{ref} \approx c_1$.

The change in the quantity of heat passing through the transverse cross section of the rod is equal to*

$$\frac{dQ}{dx}dx = -\frac{d}{dx}\left(-\lambda\frac{dT}{dx}f\right)dx, \qquad (1)$$

where f is the area of the transverse cross section and λ is the coefficient of thermal conductivity.

In the steady regime this quantity of heat is equal to

$$dQ = c_{ref} P\,[T^4 - T_2^4]\,10^{-8}\,dS, \qquad (2)$$

where P is the perimeter of the rod in the section x; $dS = (1 - 1/4(d\delta/dx)^2)^{1/2}\,dx$ is the differential of the curvilinear surface coordinate.

Thus, from (1) and (2) we have (f = const)

$$\frac{d^2T}{dx^2} = \frac{c_{ref}P}{\lambda f}\,10^{-8}(T^4 - T_2^4). \qquad (3)$$

We integrate Eq. (3) under the following boundary conditions:

$$x = 0 \qquad T = T_0; \qquad (4)$$

$$\left.\begin{aligned} &x = l \qquad T = T_l, \\ &-\lambda\left(\frac{dT}{dx}\right)_{x=l} f = c_{ref} f\,10^{-8}(T_l^4 - T_2^4). \end{aligned}\right\} \qquad (5)$$

The last indicates that all of the heat reaching the end of the rod is removed by radiation through the end surface.

Let $A = (c_{ref}P/\lambda f)\,10^{-8}$ denote some constant characterizing the geometry and material of the rod.

Double integration of Eq. (3) yields

$$\frac{dT}{dx} = \sqrt{2A\left(\frac{T^5}{5} - T_2^4 T\right) + C_1} \qquad (6)$$

and

$$\int \frac{dT}{\sqrt{2A\left(\frac{T^5}{5} - T_2^4 T\right) + C_1}} = x + C_2. \qquad (7)$$

The solution for the law governing the distribution of temperature along the rod cannot thus be found in explicit form (to take the integral in the left-hand part of Eq. (7), we must resort to series expansion). However, as will become evident from the following, this circumstance will not interfere with the attainment of a final result.

Since for $x = l$ we have from (6) that

$$\left(\frac{dT}{dx}\right)_{x=l} = \sqrt{2A\left(\frac{T_l^5}{5} - T_2^4 T_l\right) + C_1}\,, \qquad (8)$$

using condition (5), after simple transformations, we have

$$C_1 = \frac{c_{ref}^2}{\lambda^2}\,10^{-16}(T_l^4 - T_2^4) - \frac{2c_{ref}P}{\lambda f}10^{-8}\left(\frac{T_l^5}{5} - T_2^4 T_l\right). \qquad (9)$$

Since the quantity of heat given up to the ambient medium through the side surface in the steady regime

*The minus sign in the right-hand member of the equation denotes the fact that when $T > T_2$, the heat flow along the rod is reduced by the radiation from the lateral surface.

is equal to the quantity of heat entering the rod through its closed end,*

$$Q = -\lambda f \left(\frac{dT}{dx}\right)_{x=0}. \qquad (10)$$

With consideration of conditions (10) and (4), from (8) we thus obtain

$$\left(\frac{dT}{dx}\right)_{x=0} = \sqrt{2A\left(\frac{T_0^5}{5} - T_2^4 T_0\right) + C_1} = -\frac{Q}{\lambda f} \qquad (11)$$

and finally

$$C_1 = \frac{Q^2}{\lambda^2 f^2} - \frac{2c_{\text{ref}}P}{\lambda f}10^{-8}\left(\frac{T_0^5}{5} - T_2^4 T_0\right). \qquad (12)$$

Equating (9) and (12), we obtain

$$\lambda = \frac{(Q^2/f^2) - c_{\text{ref}}^2 10^{-16}(T_l^4 - T_2^4)}{2\cdot 10^{-8} c_{\text{ref}} P\left[\left(\frac{T_0^5}{5} - T_2^4 T_0\right) - \left(\frac{T_l^5}{5} - T_2^4 T_l\right)\right]} f.$$

This expression can be used for an experimental determination of the thermal conductivity λ, if we know the radiation factor c_{ref}.** However, it is best to use this method to determine the coefficient c_{ref} if λ has been found by means of some other method.

In the latter case, expression (13) should be solved for $c_{ref} \approx c_1 (F_2 \gg F_1)$.

If we assume that the radiation through the end face is negligibly small in comparison with the flow through the side surface ($T_1 = T_2$), we will correspondingly obtain

$$\lambda = \frac{Q\ 10^8}{2c_1 f P\left(\frac{T_0^5}{5} - T_2^4 T_0\right)}$$

and

$$c_1 = \frac{Q\ 10^8}{2\lambda f P\left(\frac{T_0^5}{5} - T_2^4 T_0\right)}.$$

*For a quasi-steady regime of regular heating, we must take into consideration the quantity of heat accumulated by the mass of the rod material.

**For example, assuming $c_{ref} \approx c_1$ and blackening the surface of the specimen, we will have $c_{ref} \approx c_1 = c_0 = 5.67\ \text{W/m}^2\cdot\text{deg}^4$

NOTATION

T is the temperature, °K; F is the surface, m^2; Q is the heat per unit time, W; P is the perimeter, m; f is the cross-sectional area, m^2; λ is the thermal conductivity, W/m·deg; c is the radiation factor, $W/m^2\cdot deg^4$; x is the coordinate along axis, m; l is the bar length, m; 0 is the initial section; l is the final section; 1 is the bar; 2 is the chamber wall.

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